

BE-FRAMES

REANALYSIS AND INFLUENCE FUNCTIONS

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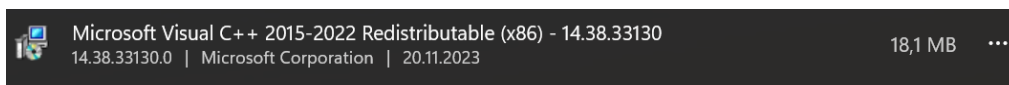
1. INSTALLATION

1.1. INSTALLATION

1. Unpack the file BE-FRAMES.ZIP and click on the file setup.exe
2. The program is installed in the folder

C:\Programs (x86)\BE-STATIK\BE-FRAMES 2.8 (or a higher version number than 2.8)

3. With the program, the Visual Studio 2022 dll is installed (download from the Microsoft server) if it is not already present on the computer. You will then see an entry for the dll in the list of installed programs.

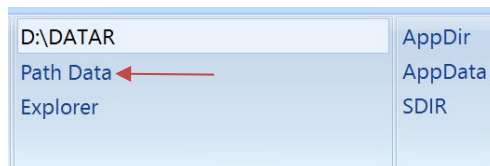


You can also install the dll separately beforehand. Download the file vc_redist.x86.exe from the Internet https://aka.ms/vs/17/release/vc_redist.x86.exe and double-click on the file.

4. The program uses a Data Path. The Data Path is the name of a folder on your computer which you specify. It serves as the 'root folder' for the subdirectories SDIR... which contain the datafiles of the single positions. After the installation, the Data Path points to the directory

C:\Users\...\AppData\Roaming\BE-FRAMES

which is an inconveniently long path, in particular, for editing chores. So, the first thing you should do is to choose a new Data Path, a shorter Data Path. For this click on the button Path Data



and choose a more convenient folder as your root folder. Either you pick an existing folder, or you enter the name of a new folder which the program will generate for you. End the input of a new folder with the Enter-key. In the example above the Data Path is D:\DATAR.

Be careful not to choose a folder to which the program has no write-access, where it cannot store files as e.g. the installation folder (not allowed by Windows)

2. INTRODUCTION

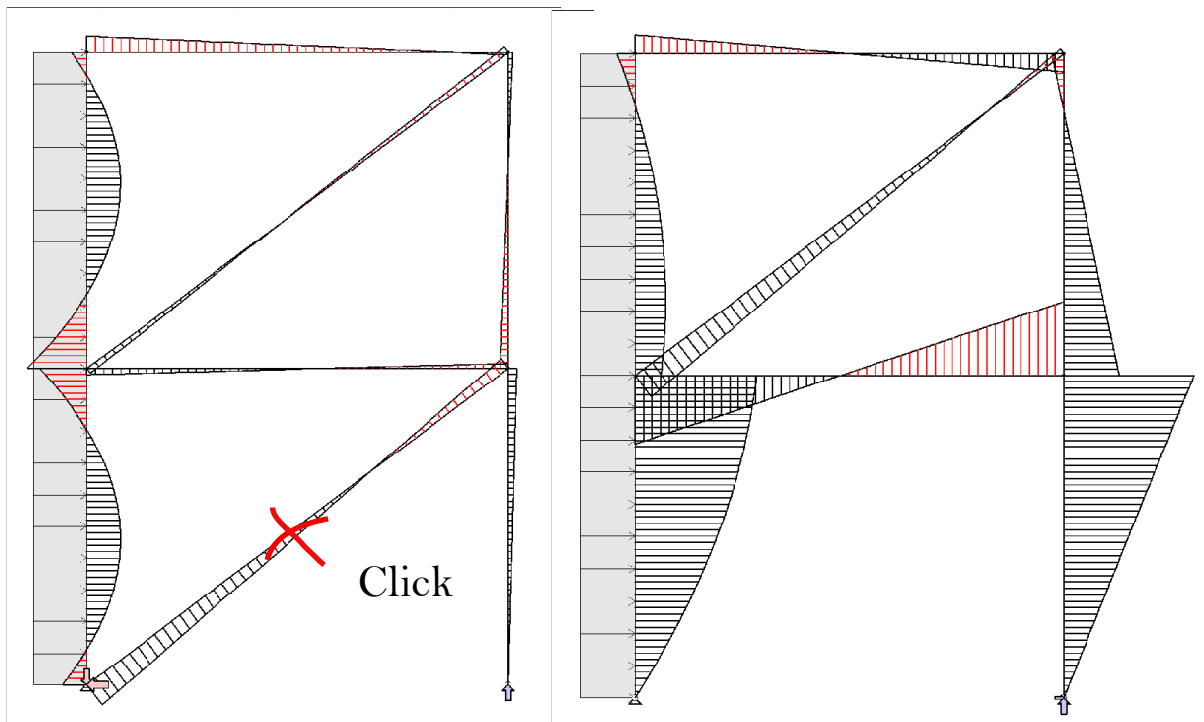
This program is a learning tool for students who want to understand:

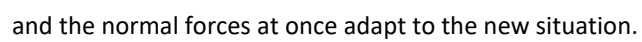
- How the design influences the load carrying capacity of a structure.
- How changes in the stiffness of single frame elements influence the distribution of the forces in a frame.
- The shapes of the different influence functions for internal forces, displacements, and support reactions in a frame.

2.1. EXAMPLES

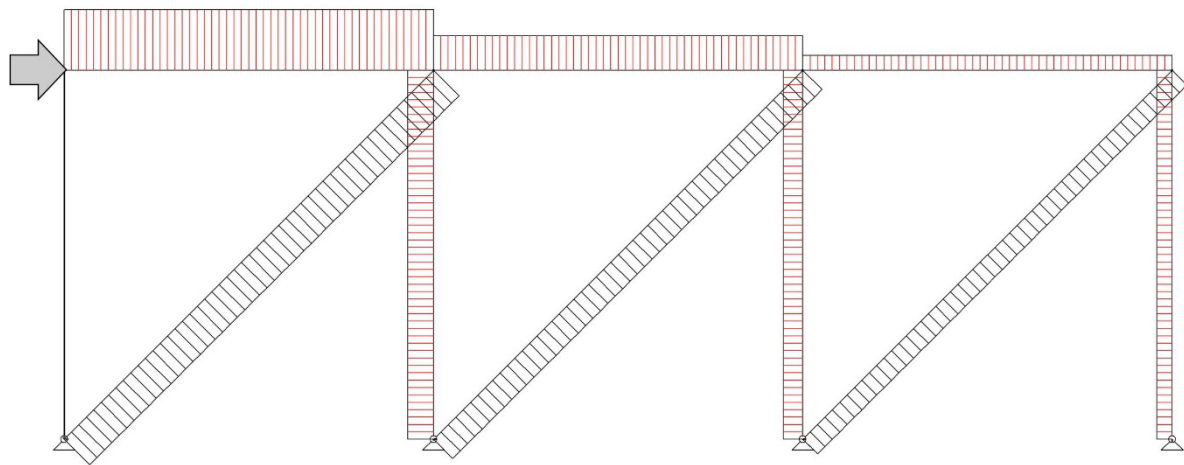
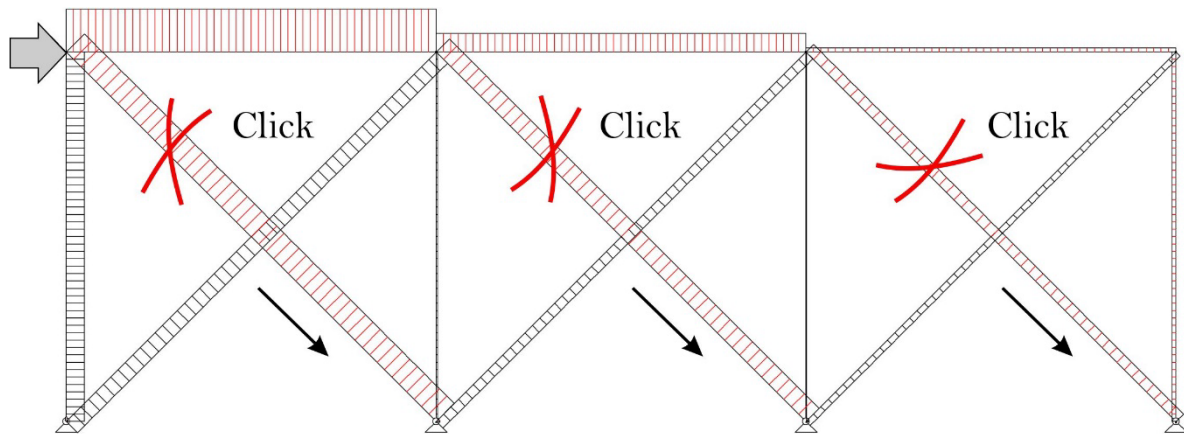
Two features characterize the program: **One-Click Reanalysis** and **Influence Functions**.

One-click reanalysis means that you can click on any element and the effects of a stiffness modification in that element immediately become visible, as in the following figure where a click on an element eliminates the element and the new bending moment distribution is at once displayed, compare left and right.

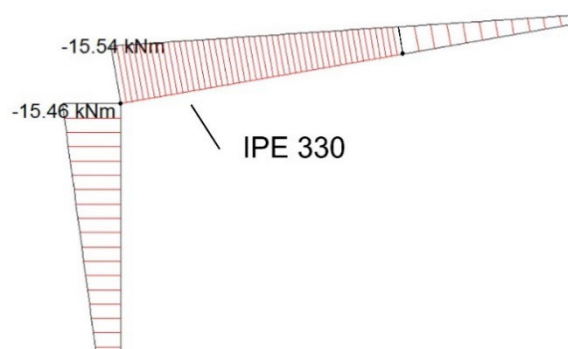
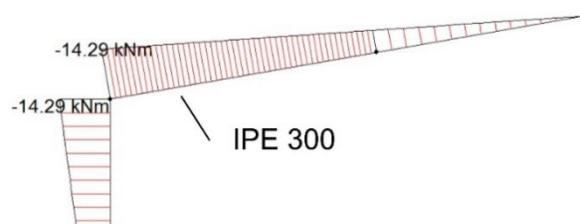




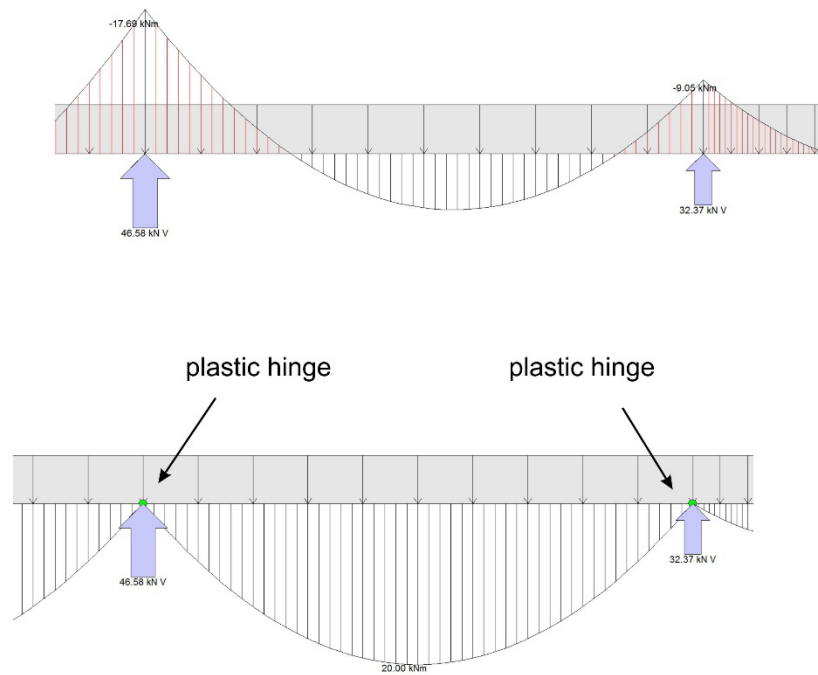
Or with three simple clicks you can eliminate the frame elements which buckle under compression



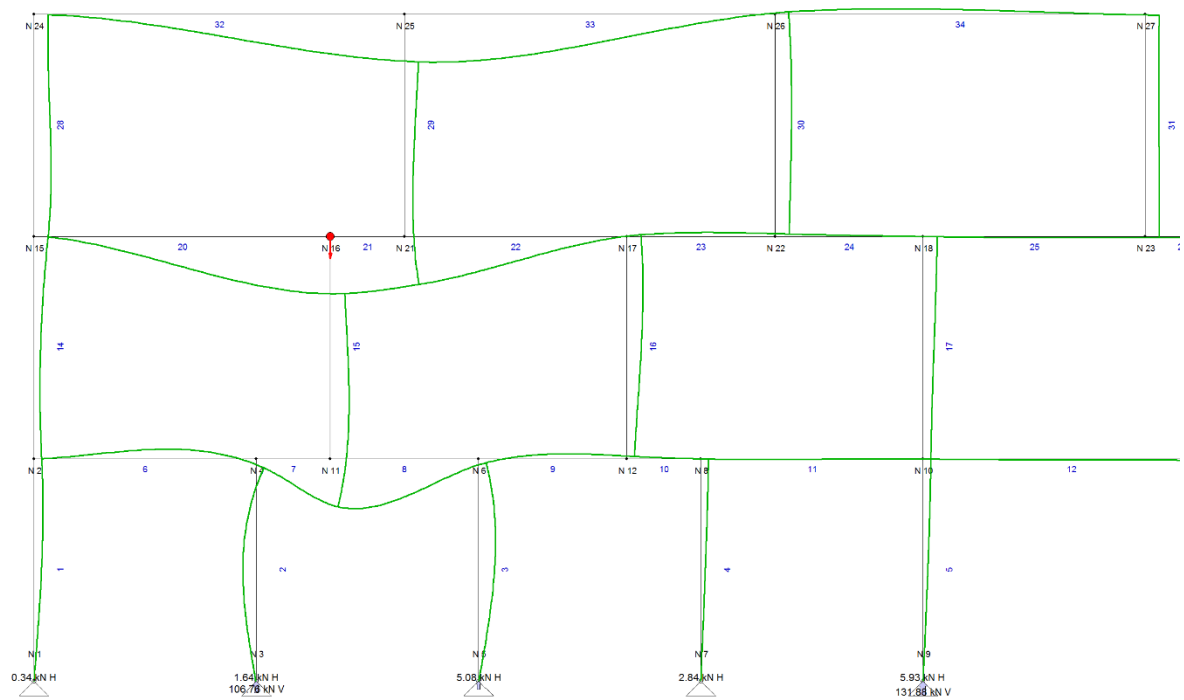
and see how the forces distribute in that situation (this can also be done automatically). Or you change the cross section of a girder to study how this effects the bending moments in the portal frame.



You can even introduce plastic hinges in a frame as in the following drawing.



In addition, the program generates the influence functions for all the standard values of a frame.



2.2. HOW REANALYSIS IS DONE

While traditionally any variation in the design requires a new build of the global stiffness matrix

$$\mathbf{K} \mathbf{u} = \mathbf{f} \quad \rightarrow \quad (\mathbf{K} + \Delta \mathbf{K}) \mathbf{u}_c = \mathbf{f}$$

to determine the new equilibrium position \mathbf{u}_c of the frame, we employ the \mathbf{f}^+ technique

$$\mathbf{K} \mathbf{u}_c = \mathbf{f} - \mathbf{K}^{(-1)} \Delta \mathbf{K} \mathbf{u}_c = \mathbf{f} + \mathbf{f}^+ \quad (\mathbf{f}^+ = -\mathbf{K}^{(-1)} \Delta \mathbf{K} \mathbf{u}_c)$$

which provides the same answer \mathbf{u}_c but keeps the original stiffness matrix intact. The new position vector \mathbf{u}_c is found by iteration

$$\mathbf{u}_c^{(i+1)} = -\mathbf{K}^{(-1)} \Delta \mathbf{K} \mathbf{u}_c^{(i)} + \mathbf{u} \quad i = 1, 2, \dots \quad \mathbf{u}_c^{(0)} = \mathbf{u}$$

which normally is very fast, so that the user gets an immediate response from the program.

Alternatively, the program finds the new \mathbf{u}_c by direct solution of only a 6 x 6 system, see Section 13. No need to solve the full system!

2.3. PROS AND CON

Pros: No need for a new build of the global stiffness matrix and no need to update the data structure. The consequences of the modifications are at once displayed on the screen.

Cons: The inverse stiffness matrix $\mathbf{K}^{(-1)}$ of the frame is needed. (It is calculated only once). One benefit though is that this allows to plot the nodal influence functions since the columns of $\mathbf{K}^{(-1)}$ are just the nodal values of these influence functions which stand for the sensitivities of a frame.

2.4. CAVEAT

This program is not for 'production', it is not a professional tool. Its main purpose is to provide students with an instrument to probe fresh design concepts with one-click reanalysis. We cannot guarantee that the results are always correct. The \mathbf{f}^+ approach itself is an approximate method. Not every decimal place will be correct.

3. HANDLING

3.1. POSITIONS

Each frame is assigned a position number

123 200 3400 7777

which can also be a string

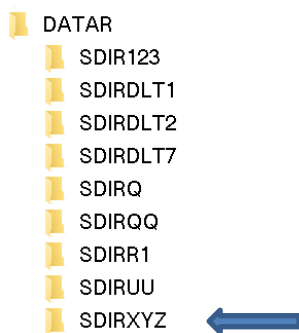
TESTABC

as long as the string does not contain special characters as in **231.2** or **A+1**.

The data of a position such as XYZ are stored in a folder

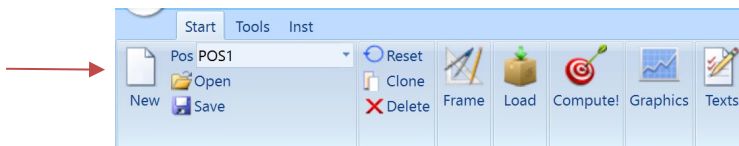
SDIRXYZ

which branches off from the Path Data (here D:\DATAR).

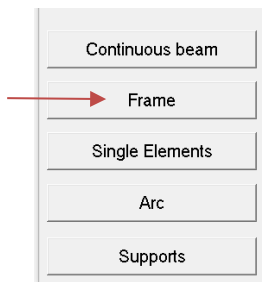


3.2. FIRST FRAME

Click on the button **New** and choose a name, say POS1, for the first frame.



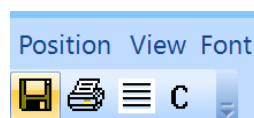
Click on the button **Frame** in the main menu and next on the button **Frame** in the program that opens



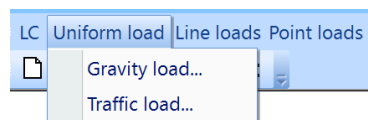
Enter the following numbers to generate a three-story frame

| Frame | | | |
|-------|------------------|-------------------|-------------|
| Floor | Floor height [m] | Number of columns | Spacing [m] |
| 1 | 3 | 4 | 4 |
| 2 | 3 | 4 | 4 |
| 3 | 3 | 4 | 4 |
| 4 | | | |

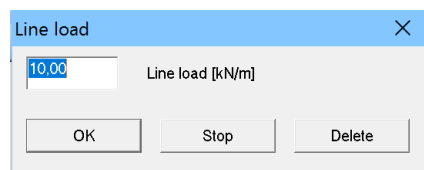
Click on the OK-button (in this dialog) and click on the disk-symbol in the tool bar to save the input



and return to the main menu of the program. Back in the main menu click on the Load button and next on Uniform load -> Traffic load



Accept the default value



and save. By a click on save (disk icon) the position is at once analyzed and the results are ready for your inspection (no need to click the Compute! Button).

Next, start the Graphics program by clicking on the button Graphics in the main menu and in the Graphics program press the N, S or M-key to display the normal forces, the shear forces, or the bending moments of position POS1. Use the mouse wheel to scale the figures and click the right mouse button to iterate through the scales

- Default: Displacements, normal forces, etc. are scaled by a turn of the mouse wheel
- 1st click: The load is scaled
- 2nd click: The support reactions are scaled
- 3rd click: Loop repeats...

REANALYSIS

C-key = remove or alter an element

Next try reanalysis. To prepare, have the bending moments or the normal forces of the frame displayed on the screen. *Press the C-key*. The cursor becomes a small square. *Click on any element*. The element is removed, and you see at once the effects. (Actually, the effects depend on the parameter k chosen. For this do a right click with the mouse). To repeat, *press again the C-key* and choose another element. To navigate forward or backward in the chain of modifications, use the *arrow-keys*, left and right, or click on the space bar. The *space bar* toggles directly between the original system and the most modified version of the system.

Press the ESC-key to reset everything to the original state. Next try the F-key.

F-key = plastic hinge $M_{\text{plastic}} = k * M$ where k = chosen parameter (right click)

The cursor turns again into a small square. *Click on any element*. Depending on where you click the element, a plastic hinge is introduced at three possible locations: left, center, right.

G-key = shear-hinge $V_{\text{plastic}} = k * V$ where k = chosen parameter (right click)

When you do the same with the G-key a shear-hinge is inserted.

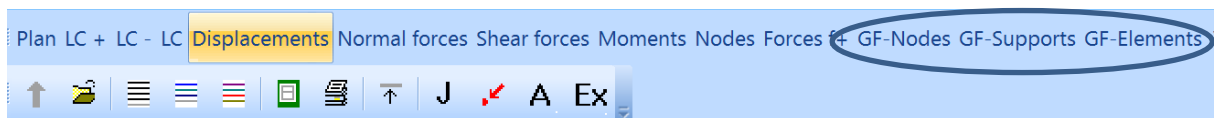
H-key = $V = M = 0$

When you hit the H-key and you click on a frame element, a shear hinge ($V = 0$) and a plastic hinge ($M = 0$) are inserted.

In the C-mode (C-key is pressed) you can also remove supports. To do this click slightly below the support. In the dialog you can specify the support stiffness in different directions. To remove the support simply click on the button

Click here to set all values k to zero = no support

When you are done with the modifications click on the ESC-key to set everything back to the original state.



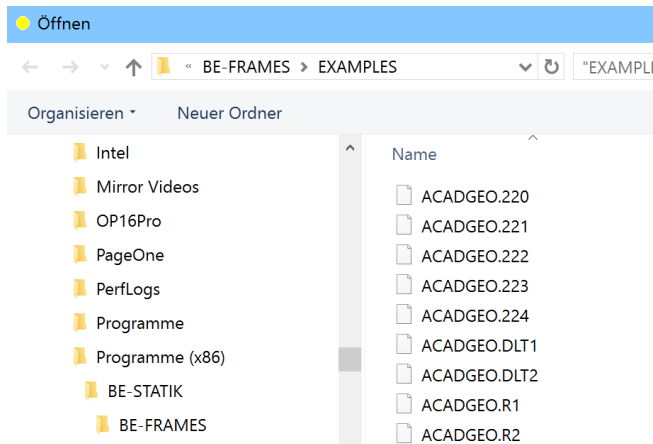
Other buttons activate the display of the Green's functions of the nodal displacements, the support reactions and of the displacements and internal actions at the mid-points of the elements. In this mode, the Up- and Down-keys let you switch between the single degrees of freedom. You can also press the right mouse button to advance to the next degree of freedom.

3.3. ADDITIONAL EXAMPLES

The program comes with a list of ready-to-install positions. To open this list, click on the button Tools and next on the button



If all is well, you see a folder EXAMPLES which contains files called ACADGEO.*.



When you double-click on any one of these files the corresponding position is automatically loaded, and you can at once start the analysis by clicking on the button Calculate!

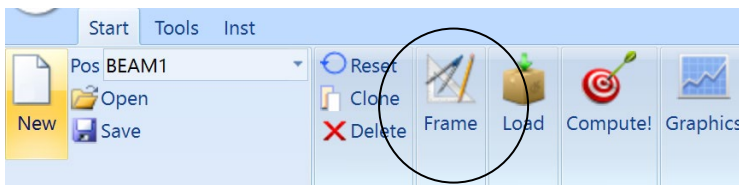
In case the folder EXAMPLES is not visible, do the following: Navigate to the folder that contains the program

C:\Program Files (x86)\BE-STATIK\BE-FRAMES

and look for the folder EXAMPLES, open the folder and pick an ACADGEO.* file. The program remembers the path from then on.

4. INPUT OF A CONTINUOUS BEAM

Start in the main menu, and enter a new position, choose for example the name BEAM1,



and click on the button Frame and then on the buttons **Material** and **Cross Sections** to specify these values

| | |
|----------------|------|
| OK | Stop |
| Title | |
| Material | |
| Cross Sections | |

Next click on the button **Continuous Beam**

| |
|-----------------|
| Continuous beam |
|-----------------|

A continuous beam with equally spaced elements is best entered in the following way:

Enter the x-coordinate of the endpoint of the first span and then press the Enter-key 3 times to generate two more spans, each with the same length.

The supports are automatically generated.

| Span | x [m] |
|------|-------|
| 1 | 4.00 |
| 2 | 8.00 |
| 3 | 12.00 |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |

x = end of span

increasing:

x = 4.0

x = 7.5

x = 12.3 ...

equally spaced

Enter span one

Return-key

Return-key

Return-key



5. INPUT OF A FRAME

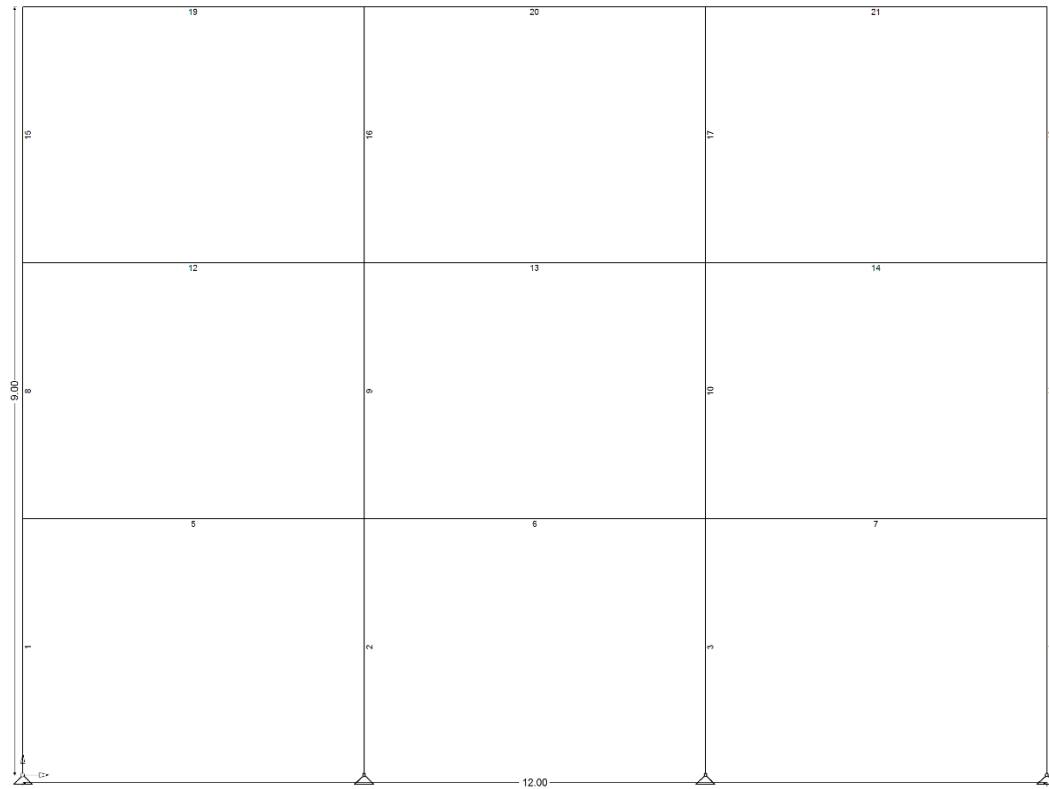
Click on the button Frame

A screenshot of a software window with a vertical menu. The menu items are: OK, Stop, Title, Material, Cross Sections, Continuous beam, **Frame**, Single Elements, Arc, and Supports. The 'Frame' option is highlighted with a darker background.

and specify the floor height, the number of columns and the spacing in between.

| Frame | | | |
|-------|------------------|-------------------|-------------|
| Floor | Floor height [m] | Number of columns | Spacing [m] |
| 1 | 3 | 4 | 4 |
| 2 | 3 | 4 | 4 |
| 3 | 3 | 4 | 4 |
| 4 | | | |
| 5 | | | |

If the second floor is a copy of the first, then simply hit the Enter-key to copy the values from the row above.



To edit the supports, click on the button Supports.

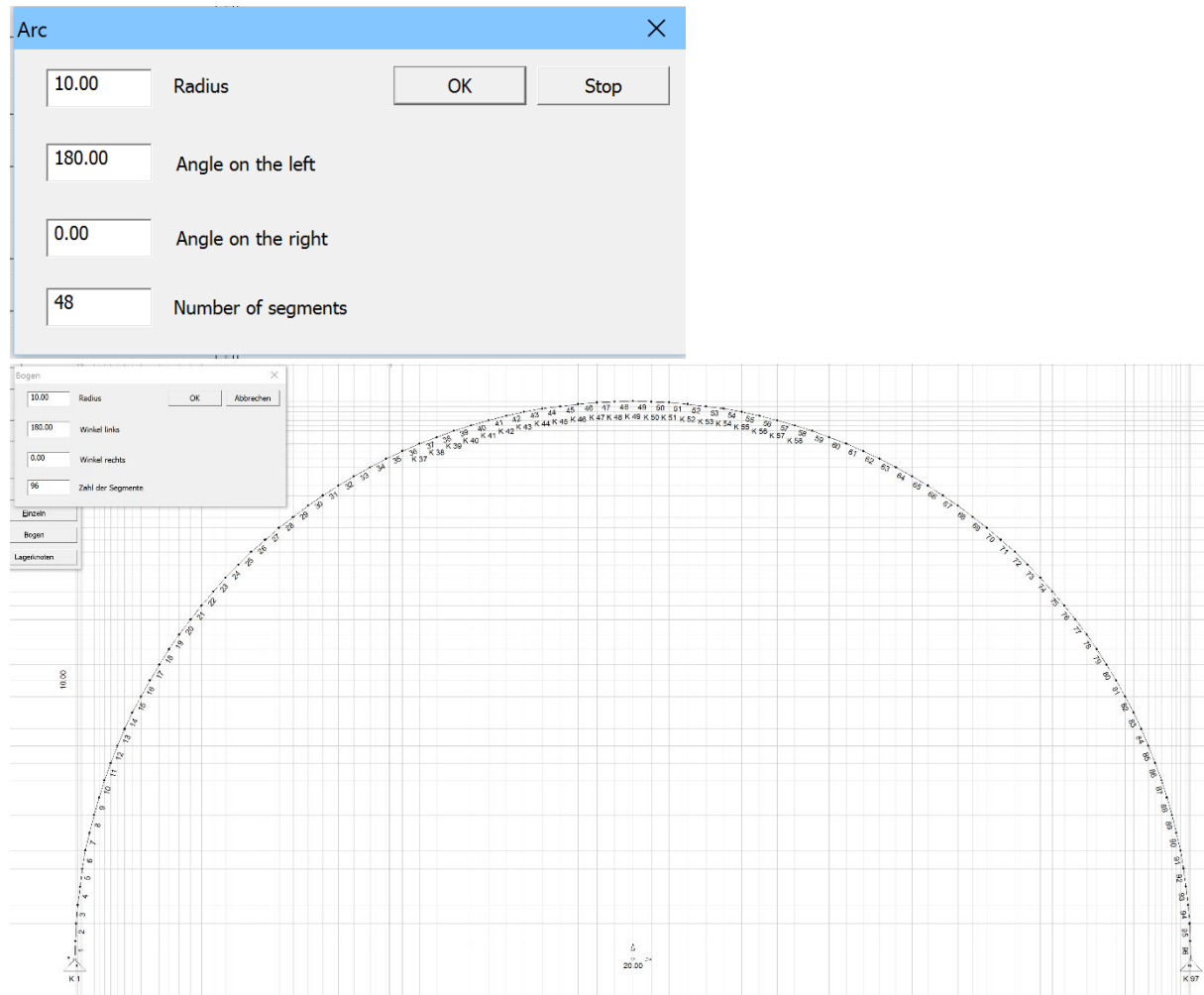
| Nodes | | | | | |
|-------|-------|------|--------|--------|--------|
| Nodes | x | y | u | v | phi |
| 1 | 0.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 2 | 0.00 | 3.00 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 4.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 4 | 4.00 | 3.00 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 8.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 6 | 8.00 | 3.00 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 12.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 8 | 12.00 | 3.00 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 0.00 | 6.00 | 0.0000 | 0.0000 | 0.0000 |
| 10 | 4.00 | 6.00 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 8.00 | 6.00 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 12.00 | 6.00 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 0.00 | 9.00 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 4.00 | 9.00 | 0.0000 | 0.0000 | 0.0000 |
| 15 | 8.00 | 9.00 | 0.0000 | 0.0000 | 0.0000 |
| 16 | 12.00 | 9.00 | 0.0000 | 0.0000 | 0.0000 |
| 17 | | | | | |

The entries in the columns u, v and phi, the values 0 to 1, represent the stiffness of the supports.

0 = free, 1 = fixed, and any value in between is allowed. The actual support stiffness of a node are these weights times the base stiffness, which (in numbers) is set to $\max E \cdot \max A$ (= max cross-section).

6. INPUT OF AN ARC

Arcs are approximated by a sequence of straight frame elements.



7. SINGLE INPUT

You can specify the elements of a frame as well single handedly, element for element.

| xa [m] | ya [m] | xb [m] | yb [m] | Mat. | Sect. | C | T | H left | H right |
|--------|--------|--------|--------|------|-------|---|---|--------|---------|
| 0.00 | 0.00 | 0.00 | 3.00 | 1 | 1 | 1 | 1 | 0 | 0 |
| 4.00 | 0.00 | 4.00 | 3.00 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0.00 | 3.00 | 4.00 | 3.00 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0.00 | 0.00 | 4.00 | 3.00 | 1 | 1 | 0 | 1 | 1 | 1 |
| 4.00 | 0.00 | 0.00 | 3.00 | 1 | 1 | 0 | 1 | 1 | 1 |

OK S P

Hinges: H (left) H (right)

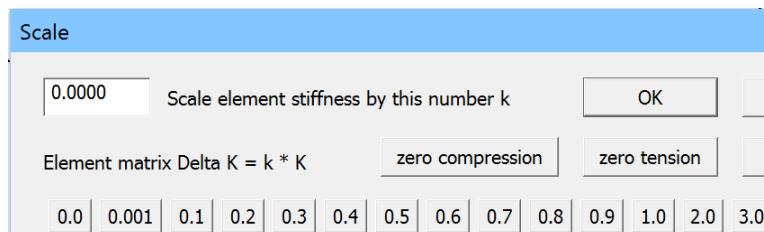
0 = No
1 = Yes

C = Compression, 0 = No, 1 = Yes
T = Tension, 0 = No, 1 = Yes

A zero in column C says that the element will buckle under compression and a zero in column T says that tensile forces are not carried by the element.

In the ground state these attributes C and T are ignored, that is all elements carry compressive as well as tensile forces.

To disable all members of type C = 0, start the program Graphics, click on the menu bar entry Parameter, and click on the button **zero compression**



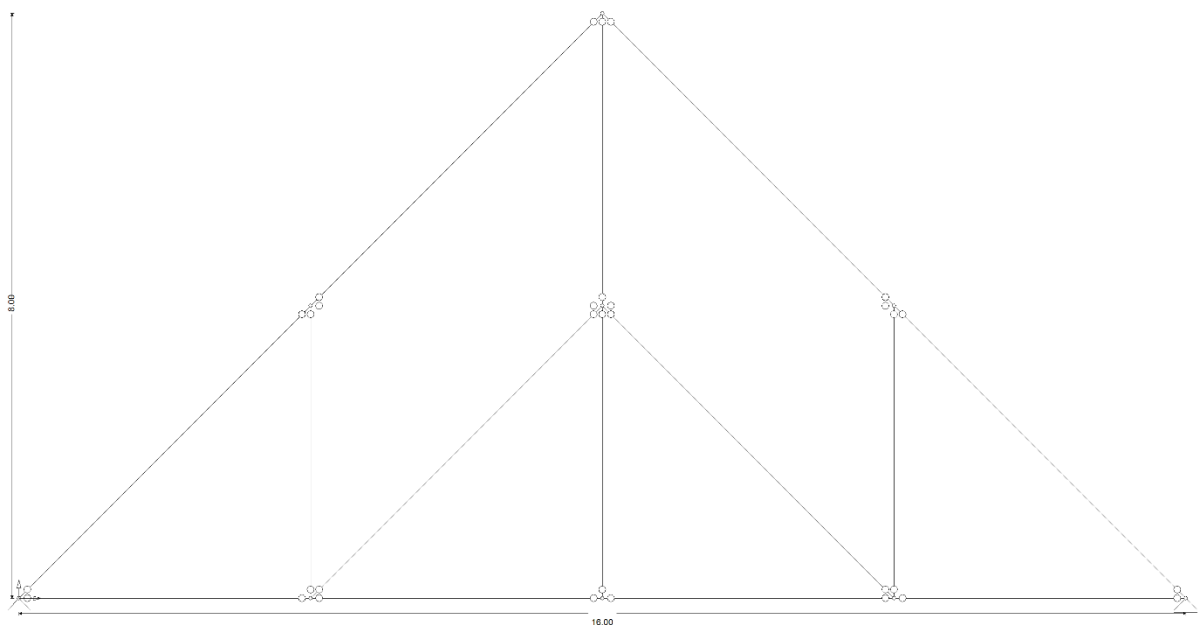
The program will then try to find iteratively an equilibrium position of the frame where all frame elements of type C = 0 are consecutively deactivated, actually are removed, if N is less than zero.

8. TRUSSES

In trusses each frame element has a hinge (H) at both ends, H left = 1, H right = 1

| Element | xa [m] | ya [m] | xb [m] | yb [m] | Mat. | Sect. | C | T | H left | H right |
|---------|--------|--------|--------|--------|------|-------|---|---|--------|---------|
| 1 | 0.00 | 0.00 | 4.00 | 0.00 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 4.00 | 0.00 | 8.00 | 0.00 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 8.00 | 0.00 | 12.00 | 0.00 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 12.00 | 0.00 | 16.00 | 0.00 | 1 | 1 | 1 | 1 | 1 | 1 |

as in the following figure



The hinges are displayed in this way, when you click on the menu entry Elevation in the Graphics program. Internally, of course, each such cluster of hinges concentrated a node is one single hinge.

9. SUPPORT STIFFNESS

The program calculates with a base stiffness.

base stiffness = (max E) x (max A) A = max cross section

All fixed supports are modeled as very stiff springs with k equal to the base stiffness.

This approach ends the need to distinguish between a reduced and a non-reduced global stiffness matrix and makes it easy to model stiffness changes in supports.

The base stiffness times the entries in the following table is the actual support stiffness of the single fixed nodes.

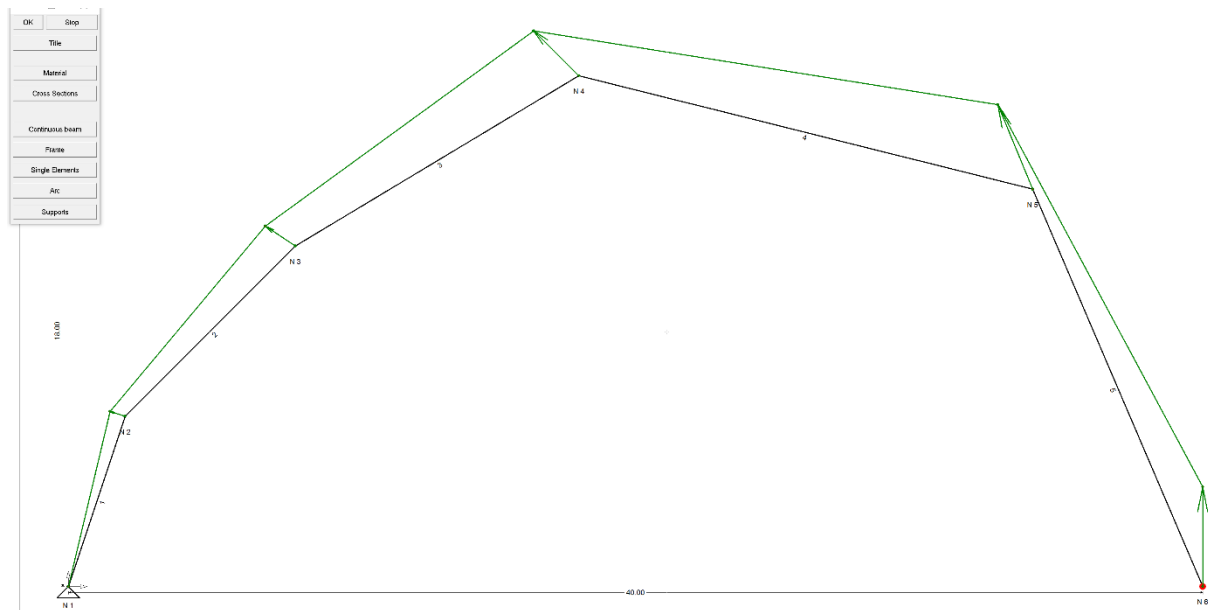
| Nodes | x | y | u | v | phi |
|-------|-------|------|--------|--------|--------|
| 1 | 0.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 2 | 0.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 3 | 4.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 4 | 4.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 5 | 8.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 6 | 8.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 7 | 12.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 8 | 12.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 9 | 16.00 | 0.00 | 1.0000 | 1.0000 | 0.0000 |
| 10 | 16.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 11 | 3.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 12 | 6.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 13 | 9.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 14 | 15.00 | 4.00 | 0.0000 | 0.0000 | 0.0000 |
| 15 | 0.00 | 8.00 | 0.0000 | 0.0000 | 0.0000 |
| 16 | 3.00 | 8.00 | 0.0000 | 0.0000 | 0.0000 |
| 17 | 6.00 | 8.00 | 0.0000 | 0.0000 | 0.0000 |
| 18 | 9.00 | 8.00 | 0.0000 | 0.0000 | 0.0000 |
| 19 | 12.00 | 8.00 | 0.0000 | 0.0000 | 0.0000 |
| 20 | 15.00 | 8.00 | 0.0000 | 0.0000 | 0.0000 |

OK Stop

0 = free, 1 = fixed
or any value in between
Table entry * base stiffness
is support stiffness

Base stiffness
2.8489E+006 kN/m (kNm)

An entry of 1.0000 means the node is fixed and an entry 0.0000 means that the node is unrestrained. Any other values in between are also possible.



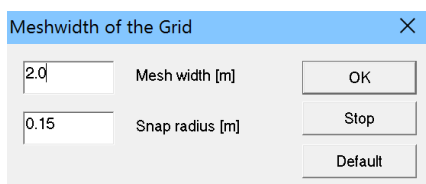
If the system is kinematic because a support is missing or because it has a hinge too many the program flags this as an error and displays the possible rigid-body movement as in the figure above where the missing vertical support at the right end allows the frame to rotate about the left support.

10. GRAPHICAL INPUT

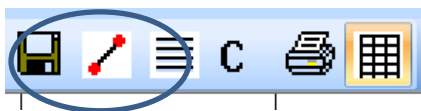
You can also draw a frame with the mouse. First click on the Grid-button



and choose a suitable mesh width.



To activate the drawing mode, click on the line-icon.



and then click on the nodes of the grid to outline the shape of the frame. (You must not use the grid).

To pause, press the spacebar and to resume press the spacebar again. To stop the input mode, click on ESC.

To finalize the drawing click on the button Single Elements and if everything is correct specify the supports by clicking on the button Supports.



Note that at the end of the drawing mode you **must** click on the Single Elements dialog. This way the program has a chance to understand the layout of the frame.

11. FINITE ELEMENTS 'PURE'

A click on the icon



displays the equivalent nodal forces which belong to the actual load case. The reaction of the structure to these nodal forces is the FE-solution 'pure'. The nodal values of this solution are the entries of the vector \mathbf{u} which solves the system $\mathbf{K} \mathbf{u} = \mathbf{f}$.

The complete solution is the FE-solution 'pure' plus the local solutions in the single frame elements.

- Complete solution = FE-solution 'pure' + local solutions (not displayed when key is pressed)

The local solutions are the element solutions when the frame elements are clamped at both ends.

To display the complete solution simply turn the FE-key off.

12. WHEN RESULTS ARE EXACT

Point values are only exact if

- the Green's function of that point value lies in V_h or
- the error in the Green's function is orthogonal to the load.

The second condition is satisfied if the exact solution lies in V_h .

Consider a simple hinged beam with a uniform load p and subdivided into four elements. The FE-solution is exact also in between the nodes - but why?

- The exact solution is a 4th order polynomial which does not lie in V_h because all shape functions are 3rd order polynomials.
- The Green's functions for point values, say $w(x)$, in between nodes do not lie in V_h .

So theoretically point values in between the nodes should not be exact. The trick is that the program adds to the pure FE-solution (only equivalent nodal forces) the local solutions in the elements.

The point values of the pure FE-solution are not exact, but the values of the complete solution are

- Complete solution = FE-solution 'pure' + local solution = exact solution

Note: In all elements which are load-free the local solution is zero (no need for a clamped element to deflect when there is no load) and in these elements the pure FE-solution is the complete, the exact solution.

13. EXAMPLE

An illustrative example of the difference between the pure FE-solution and the complete solution supplies the next picture. In Figure a you see the exact influence function for the deflection at the mid-point of the first element.

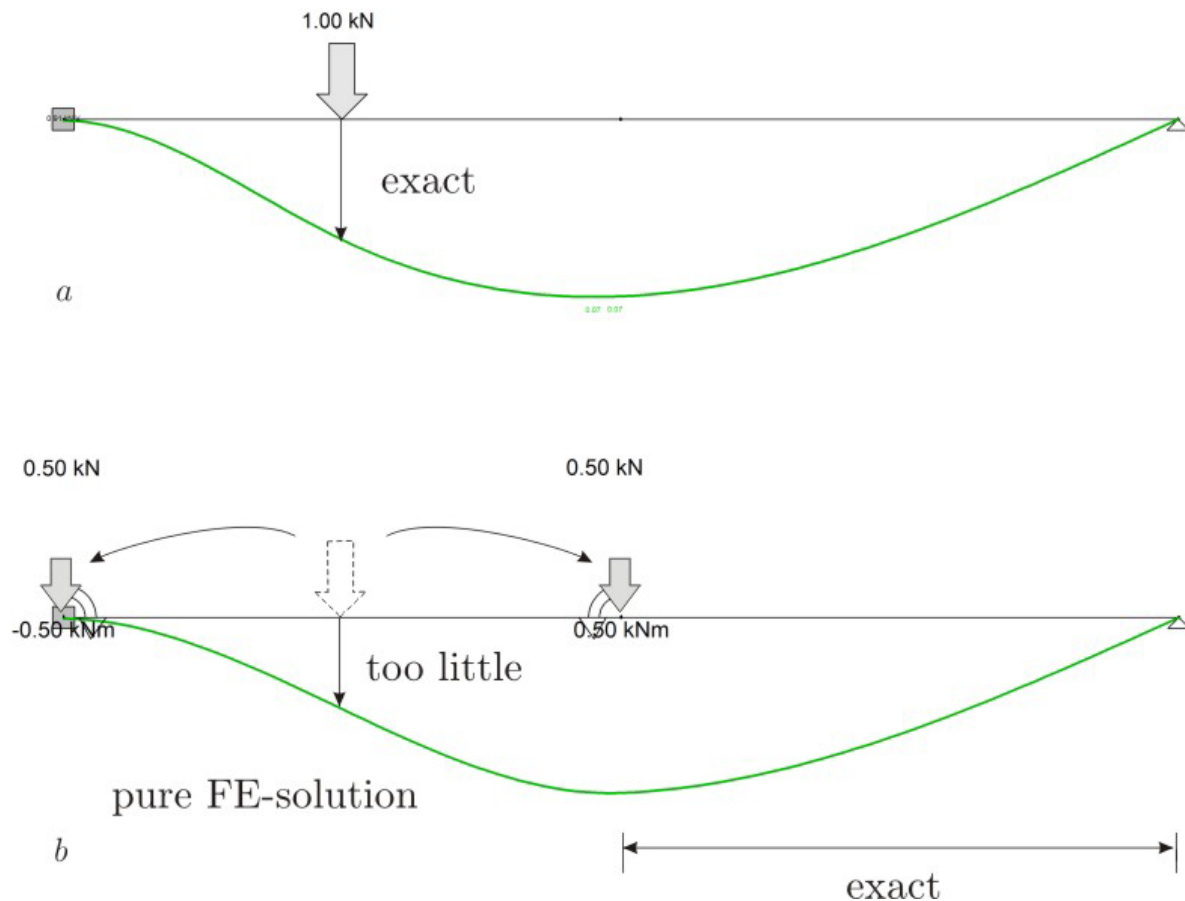
The FE-program cannot solve this load case and so it replaces the point load by equivalent nodal forces, see Figure b. The result is an influence function which – in the first element - is only an approximation.

Outside the element the FE-solution is exact because the exact influence function G is outside of the element a homogeneous solution, $EI G^{IV} = 0$, a cubic, and cubics lie in V_h .

To turn this 'defect' on you must click on the FE-button. Otherwise, the program displays the complete solution which is of course exact because the program adds the local solution to the pure FE-solution.

The local solution is the response of a clamped one-element beam loaded with a unit point load at its center.

In frame analysis, it is possible to add the local solutions to the pure FE-solution but in 2D and 3D problems this is not possible because the local solutions are unknown and because the shape functions are not homogeneous solutions of the governing equation. In 2D and 3D problems the FE-solution is the pure FE-solution.



14. REANALYSIS

Reanalysis is done in one click.

14.1. THE C-KEY

It is best to have the normal forces or the bending moments on display when you start the reanalysis. You begin by specifying the scale, by how much the stiffness of a clicked element should change. For dramatic effects, it is best to set the scale k to zero (if the frame is not statically determinate).

In the menu bar click on Parameter or simply press the right mouse button. This opens the following dialog.

Scale [X]

0.0000 Scale element matrix by this number k [OK] [Stop]

Matrix Delta $K_e = (k-1) * K_e$ [Zero compression] [Zero tension] [Help]

0.0 0.001 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 k

$K_e \text{ new} = K_e + \text{Delta } K_e = k * K_e$

In statically determinate system choose $k > 0.1$

☒ Iterative Solution ☐ Direct Solution

Iteration parameters (normally no need to edit!) [Default]

1.000E-007 Error margin, $\epsilon = 1.0E-5$ (etc.)

100000 Max Iterations

where you can specify the scale k. Set k equal to zero. Close the dialog and then hit the C-key (you are then in 'C-mode') and next click on the frame element you want to remove. To repeat, hit again the C-key and click on the next element.

With the cursor keys, \leftarrow and \rightarrow , you can switch between the different modifications. The space bar toggles between the original frame and the most modified version of the frame.

To remove a support, click on the support (a little below center) and in the dialog.

Stiffness of the support [X]

2.848E+006 Base stiffness [OK] [Stop]

Values of k

1.000000 in x-direction

1.000000 in y-direction

0.000000 in turning direction

The effective support stiffness is the product of the table entries k times the base stiffness

true $k = \text{base stiffness} * k$

The factor k scales the stiffness of the support

0 = no support, 1 = rigid support

Rigid supports are very stiff springs with a stiffness equal to the base stiffness, so choose $k = 1$ for rigid supports

To model an elastic support the entries must be very small, $k = 0.001$

The following tool finds the scale k

[] k

[] target value

Click here to set k to zero in all directions = no support

copy value k to ☐ x-direction ☐ y-direction ☐ turning direction

click on

Click here to set k to zero in all directions = no support

Otherwise, you can choose any value between 0 and 1 to model a support stiffness.

The program finds the new equilibrium position of the frame either

- by an iterative analysis. The entry in the fields Error margin and Max Iterations (see above) serve to steer the iteration, or
- by direct solution

In the direct solution approach, the program solves an auxiliary 6 x 6 system, and the solution of this small system suffices to determine the new equilibrium position of the frame, regardless of how many degrees of freedom the frame has.

14.2. THE F-KEY

When you press the F-key the program is ready to place a plastic hinge ($M = 0$) in any frame element at three possible spots, left, center or right, depending on where you click on the element.

To repeat, press the F-key anew and click on the next element. In this mode, it is best to have the bending moments on display or the displacements to see the kinks at the plastic hinges.

Actually, the 'degree' of the plastic hinge can be scaled by choosing a suitable value of k in the following dialog.

An entry $k = 0$ corresponds to a true plastic hinge ($M = 0$) while an entry $k = 0.8$ means a reduction of the bending moment by 20 %, that is $M_c = M * k$. What you see on the screen then are the bending moments in the frame given that the load carrying capacity of the plastic hinge is limited to $M * k$. In this situation, the factor k only affects the capacity of the plastic hinge. Element matrices are not changed.

14.3. THE G-KEY

The G-key works the same way it is only that a shear hinge ($V = 0$) is installed in the frame element.

14.4. THE H-KEY

The H-key finally is a combination of an M-hinge and a V-hinge, that is the frame element is (not exactly) cut in two parts. Not exactly because normal forces can pass through. Only one such combo-hinge can be placed on the frame at any time, and it is always placed in the middle of the frame element (no mathematical reason for this). The scale k is automatically set to zero, $M = V = 0$.

15. INFLUENCE FUNCTIONS (OR GREEN'S FUNCTIONS)

The program can display the Green's functions for

- The nodal displacements;
- The support reactions
- The displacements and internal actions at the mid-points of the elements or anywhere else

GF-Nodes GF-Supports GF-Elements GF-Anywhere

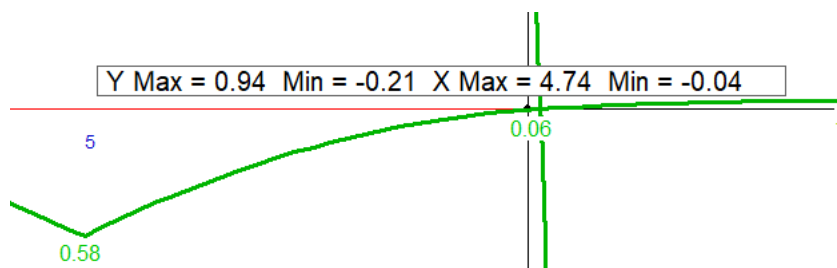
- The Green's function of a node is the displacement figure of the frame if a force $P = 1$ or a moment $M = 1$ pushes the node in the direction of one of the three degrees of freedom u_1 , u_2 or u_3 .
- At supports the Green's function is the influence function for the chosen support reaction.
- The element Green's functions are the influence functions for u , w , w' , N , V , M at the mid-point of the element.

With the **Up**- and **Down**-keys you can advance to the next Green's function. The same effect has a press on the right mouse button.

GF-Anywhere comes with the option to display the integral of the influence function.

☒ Integral value

if all elements carry a horizontal or vertical unit load $p = 1$ [kN/m]



Y Max = max value and vert. load, X Max = max value and horiz. load, Min are the min values under Y and X load.

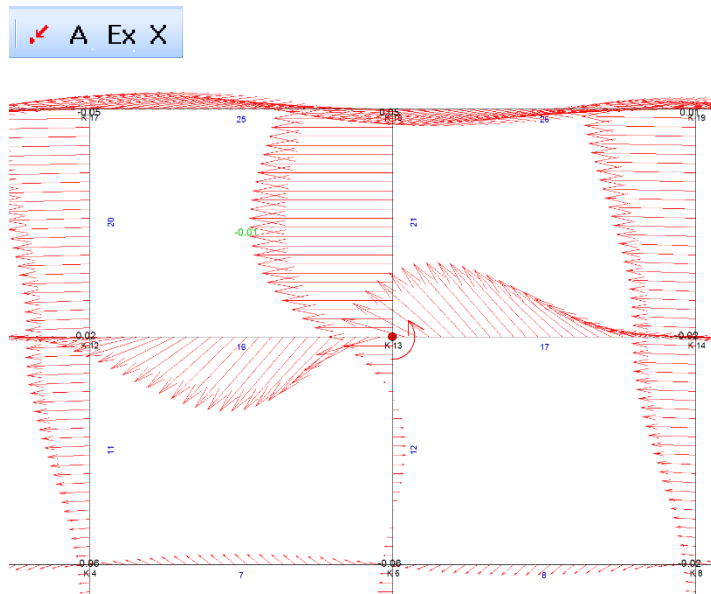
When an element Green's function is displayed, here of a normal force N at the center of the element, a text as

GF N [kN], H = 0.691 V = 0.795 R = 0.004 (H is result if all horizontal $f_i = 1$, V = result if all vertical $f_i = 1$, etc.)

is displayed. It is $H = 0.691$ the normal force N in the element when all nodes carry a unit nodal force $f_j = 1$ pointing in horizontal direction, $V = 0.795$ is the normal force N when all nodes carry unit nodal forces $f_j = 1$ pointing in vertical direction and $R = 0.004$ is the value of N if the nodes carry moments $f_j = 1$.

15.1. ARROWS

A click on the button A adds arrows to the display of the Green's functions to better visualize the character of the function



A single point load has maximum effect if it points in these directions. The length of the arrow is proportional to the influence the point load has, in the picture above on the rotation of the center node.

16. EIGENVECTORS AND EIGENVALUES

With MatLab you can calculate the eigenvectors and eigenvalues of the frame.

This is how it is done: Copy the file frameeigen.m from the program directory of BE-FRAMES to your MatLab folder (where you current MatLab projects reside).

Open Matlab and open the file

frameeigen.m

Edit the first two lines, that is enter the so called PathData and the name of the position.

Run the file.

Start the program and in graphics click on the menu entry Eigen. With the **Up**- and **Down**-keys you can switch between the eigenvectors.

(The file frameeigenEx.mlx in the program directory is a lengthier version with more comments interspersed.)

The file frameeigen.m

```

DataPath = 'd:\dat'; % <----- specify the PathData = name of the folder from
which the folder SDIR... branches off
NamePosition = 'cbeam'; % <----- specify the name of the position

path = strcat(DataPath, '\sdir', NamePosition, '\matrix.', NamePosition);

% With the above input the path points to the file
% d:\dat\sdircbeam\matrix.cbeam
%
% If the name of the position is abc and the folder sdirabc lies in the
% folder
% E:\FrameData then DataPath = 'E:\FrameData' and NamePosition = abc'
%
% path = 'E:\FrameData\sdirabc\matrix.abc'

fid = fopen(path);
VectorM = fread(fid, 'double'); % column vector
fclose(fid);

[row,col] = size(VectorM); % col = 1

n = 0.5 + sqrt(0.25 + row) - 1; n = cast(n, 'int32'); % n = number of columns

K = zeros(n+1,n); % the stiffness matrix with an extra row at the top

for i = 1:n
    K(:,i) = VectorM(1 + (i-1) * (n+1) : 1 + (i-1) * (n+1) + n);
end

K(1,:) = []; % this matrix has now size n x n and is the original stiffness matrix

[V,P] = eig(K); % all eigenvalues must be positive

path1 = strcat(DataPath, '\sdir', NamePosition, '\eigv.', NamePosition);
fid = fopen(path1, 'wb'); % eigenvalues
fwrite(fid, diag(P), 'double');
fclose(fid);

path2 = strcat(DataPath, '\sdir', NamePosition, '\ev.', NamePosition);
fid = fopen(path2, 'wb'); % eigenvectors
for i = 1:n
    fwrite(fid, V(:,i), 'double');
end
fclose(fid);

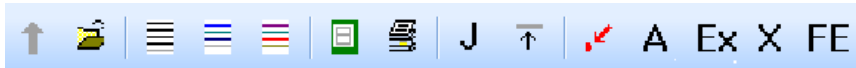
```

17. KEYS AND MOUSE

17.1. DISPLAY RESULTS

- Press N, M, S, D-key to display Normal forces, Moments etc.
- Strg + click on an element displays single values.
- To display the values along one single element, press the P-key and then click on the element.
- X-key, only horizontal displacements
- Y-key, only vertical displacements
- The mouse wheel acts on
 1. N, M, S, D
 2. The load
 3. The support reaction, in this order
- Click right mouse button to iterate through this list.

17.2. TOOLBAR



- A click on one of the first three icons loads text files (dimensions, load cases, results)
- Print preview (green icon)
- Print
- Reset displaced texts (after you have moved the texts with the mouse)
- J = jpeg
- Red arrow resets scales
- A is for arrows on the influence functions.
- Ex = exponentials, scientific notation
- X = reminder to reset the frame after the C-mode or any other mode
- FE = 'Pure FE-Results', when the local element solutions are not added

17.3. WHEEL



- Wheel = scale drawing,
- Shift + Wheel = scale size of the frame,
- Ctrl + Wheel = font size
- Alt + Wheel = line width

To zoom, first press the Z-key, and then draw a rectangle with the mouse, ESC-key = zoom off

17.4. MULTI-CLICK AND MOUSE WHEEL

At the start the mouse wheel acts as a scale on the displacements, the normal forces, etc.

1st click: The load is scaled.

2nd click: The support reactions are scaled.

3rd click: Loop repeats...

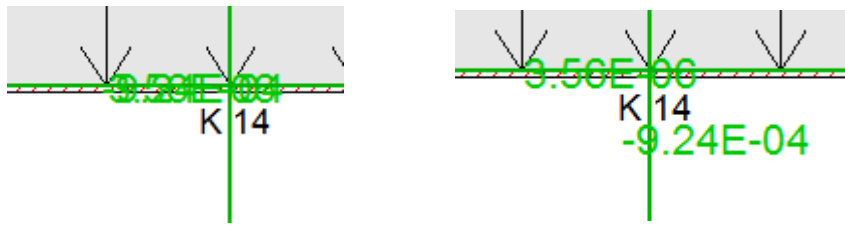
17.5. MOUSE CLICKS

When influence functions are displayed: A **Right click** advances to the next influence function. The Up- and Down-keys have the same effect.

To scale the support reactions, click near any support and rotate the mouse wheel. Same as A + Wheel. Or use the multi-click technique.

To scale the load, click on any element and turn the mouse wheel. Same as Ctrl + Wheel. Or use multi-click.

Most numbers on the screen can be moved around. Click on the number and pull it sideways. This helps to untangle messy heaps of numbers.



17.6. KEYS

- D = Displacements
- E = Element numbers on/off
- F = F-mode, plastic hinge ($M = 0$)
- G = G-mode, shear hinge ($V = 0$)
- H = H-mode, element is cut in half, $M = V = 0$
- K = Node numbers on/off
- N = Normal forces
- M = Moments,
- P = Activates 'click-mode' to display N, M, S, D of single elements by clicking on the elements
- S = Shear forces
- Strg-key + click on an element displays single value of N, M, S, D at that point.
- W = energy $\frac{1}{2} \mathbf{u}^T \mathbf{K} \mathbf{u} * 10\,000$ of single elements, button toggles
- X = only results in horizontal direction, on/off
- Y = only results in vertical direction, on/off
- Z = Numbers, on/off
- Space key = Switch between original structure and modified structure

Cursor-keys (Up and Down) = Change load case

Cursor-keys (Left and Right) = Move forward or backward in a series of modifications

17.7. HORIZONTAL DISPLACEMENT

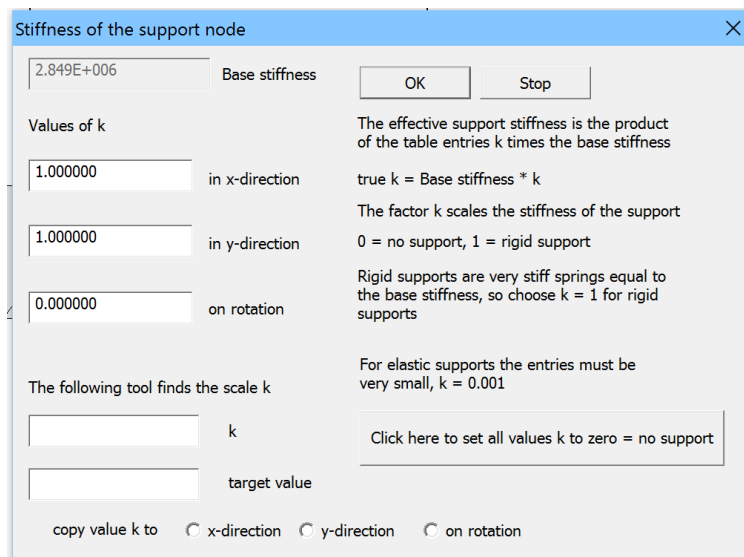
To display the horizontal displacement of a horizontal frame element, proceed as follows:
Press the X-key, press the P-key and click on the frame element.

17.8. MODIFYING ELEMENTS (C-MODE)

- Press the C-key and then click on the element which is to be removed or modified.
- Repeat!
- Traverse the sequence of modifications with the Cursor-keys (Left and Right)
- Space key = Switch between original structure and the modified structure (deepest level)
- ESC-key leaves the C-Mode

17.9. MODIFYING SUPPORTS (C-MODE)

- Press the C-key to activate the C-Mode and click on the support (a little below the support)
- Edit the stiffness of the support by choosing a suitable value for k.



Stiffness of the support node

Base stiffness: 2.849E+006

Values of k:

- 1.000000 in x-direction
- 1.000000 in y-direction
- 0.000000 on rotation

The effective support stiffness is the product of the table entries k times the base stiffness

true k = Base stiffness * k

The factor k scales the stiffness of the support

0 = no support, 1 = rigid support

Rigid supports are very stiff springs equal to the base stiffness, so choose k = 1 for rigid supports

For elastic supports the entries must be very small, k = 0.001

The following tool finds the scale k

k: [] target value: []

Click here to set all values k to zero = no support

copy value k to: ☐ x-direction ☐ y-direction ☐ on rotation

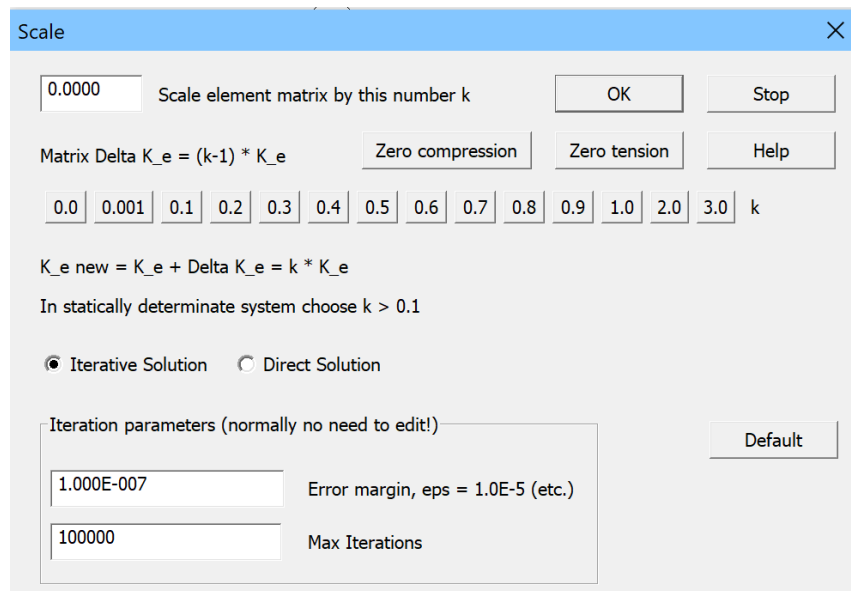
- **ESC-key** leaves the C-Mode

17.10. FINETUNING OF THE C-MODE

A click on the entry Parameter in the menu bar, or simply a click of the right mouse button,

View Font Help **Parameter Scales**

opens the following dialog



Scale

0.0000 Scale element matrix by this number k

Matrix Delta $K_e = (k-1) * K_e$

Zero compression Zero tension Help

0.0 0.001 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 2.0 3.0 k

$K_e \text{ new} = K_e + \Delta K_e = k * K_e$

In statically determinate system choose $k > 0.1$

☒ Iterative Solution ☐ Direct Solution

Iteration parameters (normally no need to edit!)

1.000E-007 Error margin, eps = 1.0E-5 (etc.)

100000 Max Iterations

Default

where you can specify the element stiffness in relation to the standard stiffness of the element.

A change in the stiffness of an element means that the element stiffness matrix is scaled with the factor k. It is not possible to scale EA or EI separately.

17.11. ZERO COMPRESSION OR ZERO TENSION

Frame members which cannot carry any compression or tension get the attribute

$$C = 0 \quad \text{or} \quad T = 0$$

In the ground state the program ignores these specifics and analyzes the frame as if all elements can carry compressive as well as tensile forces.

When you click in the above dialog on the button

Zero compression

the program will try to find iteratively (or directly, button direct solution) an equilibrium position of the frame where all frame elements of type $C = 0$ are deactivated if N is less than zero. If $k = 0$ the rule is enforced strictly, otherwise it can be attenuated by choosing a small positive factor $k > 0$. The technique is based on the forces \mathbf{f}^* .

17.12. FORCES \mathbf{F}^+

In the C-mode you can display the forces \mathbf{f}^* . These are the forces which when they act on the original frame produce the same effects as the modifications in the C-mode. To scale the forces, turn the mouse wheel. To scale the moments, press also the CTRL-key.

18. HOW IT IS DONE

When an element is removed, or altered, the program finds the new equilibrium position \mathbf{u}_c of the frame by iteration.

$$\mathbf{u}_c^{(i+1)} = -\mathbf{K}^{(-1)} \Delta \mathbf{K} \mathbf{u}_c^{(i)} + \mathbf{u} \quad i = 1, 2, \dots \quad \mathbf{u}_c^0 = \mathbf{u}$$

where $\Delta \mathbf{K}$ is the modified element stiffness matrix and $\mathbf{K}^{(-1)}$ is the inverse of the global stiffness matrix, see [1] p. 282. The matrix $\Delta \mathbf{K}$ has as many columns and rows as \mathbf{K} but it contains only the entries of the modified 6 x 6 element matrix $\Delta \mathbf{K}_e$ of the element that is removed or altered.

Alternatively, the program calculates the new vector \mathbf{u}_c directly, by solving the system

$$(\mathbf{I} + \mathbf{K}^{-1} \Delta \mathbf{K}) \mathbf{u}_c = \mathbf{u}$$

Because $\Delta \mathbf{K}$ is sparse the program must only solve a (6 x 6)-system when one single element is modified or removed. No need to solve the full system. For two elements, it would be a (12 x 12) system, etc.

When you would apply the forces

$$\mathbf{f}^+ = \Delta \mathbf{K} \mathbf{u}_c$$

to the original structure then the equilibrium position would exactly come out as the vector \mathbf{u}_c .

$$\mathbf{K} \mathbf{u}_c = \mathbf{f} + \mathbf{f}^+$$

The nodal influence functions are identical with the columns of the inverse stiffness matrix, or to be precise, the nodal values of these influence functions are the columns of $\mathbf{K}^{(-1)}$.

To calculate the element influence functions the program applies the corresponding Dirac deltas at the mid-points of the elements, that is a point force $P = 1$ in x- and y-direction, a moment $M = 1$, etc.

A plastic hinge is generated by applying a Dirac delta δ_2 (= influence function for M), that is an artificial kink and by then sizing the Dirac delta in such a way that the sum of the bending moment of the load and of the Dirac delta is zero.

The same technique is applied to generate a shear hinge with a Dirac delta δ_3 (= influence function for V) and to cut a frame element into two halves, two Dirac deltas δ_2 and δ_3 are applied simultaneously.

18.1. LIMITS

Theoretically, the iteration does not converge if the modified frame is kinematic, that is if its stiffness matrix

$$\mathbf{K}_c = \mathbf{K} + \Delta \mathbf{K}$$

is singular. This can happen when a structure is statically determinate and an element is removed, in which case $\Delta \mathbf{K} = -\mathbf{K}_e$ where \mathbf{K}_e is the element stiffness matrix, so that $\mathbf{K}_c = \mathbf{K} - \mathbf{K}_e$. To see this, we argue as follows: Because the new \mathbf{K}_c is singular there exists a non-trivial vector \mathbf{u}_0 so that

$$(\mathbf{K} + \Delta \mathbf{K}) \mathbf{u}_0 = \mathbf{0}$$

Multiplying this equation from the left with the inverse \mathbf{K}^{-1} gives

$$\mathbf{K}^{-1}(\mathbf{K} + \Delta\mathbf{K}) \mathbf{u}_0 = (\mathbf{I} - \mathbf{K}^{-1} \Delta\mathbf{K}) \mathbf{u}_0 = \mathbf{0}$$

which means that \mathbf{u}_0 is an eigenvector of $\mathbf{K}^{-1} \Delta\mathbf{K}$ with eigenvalue 1 and the consequence is that the error $\mathbf{e}_{i+1} = \mathbf{u}_{i+1} - \mathbf{u}_i$ cannot decrease

$$\mathbf{e}_{i+1} = -\mathbf{K}^{-1} \Delta\mathbf{K} \mathbf{e}_i \quad i = 1, 2, \dots$$

because such a fixed-point iteration only succeeds if the absolute values of the eigenvalues of the iteration matrix are less than one, [2] p. 564.

Note

$$\mathbf{e}_{i+1} = \mathbf{u}_{i+1} - \mathbf{u}_i = -\mathbf{K}^{-1} \Delta\mathbf{K} \mathbf{u}_i + \mathbf{u} + \mathbf{K}^{-1} \Delta\mathbf{K} \mathbf{u}_{i-1} - \mathbf{u} = -\mathbf{K}^{-1} \Delta\mathbf{K} (\mathbf{u}_i - \mathbf{u}_{i-1}) = -\mathbf{K}^{-1} \Delta\mathbf{K} \mathbf{e}_i$$

A curious point is that under mild circumstances the iteration will converge even if the removal of an element(s) has made the structure unstable because the program iterates and does not try to calculate the non-existing inverse of the singular matrix $\mathbf{K} + \Delta\mathbf{K}$. The results are seemingly the same as if you would substitute for the inverse $(\mathbf{K} + \Delta\mathbf{K})^{-1}$ the pseudoinverse.

18.2. NOT IMPLEMENTED

Reanalysis is not implemented for load cases where supports settle. Technically such load cases pose no additional problems. The reanalysis must focus on the change in the support stiffness of the node. To this end a unit force $f_i = 1$ is applied at the support and a reanalysis is done. The inverse of the support displacement caused by the unit force is the new support stiffness and with this value the necessary force can be calculated to displace the support.

Repeating this at each node one could theoretically calculate the inverse \mathbf{K}_c^{-1} of the modified structure step by step.

19. MATH

The program solves $\mathbf{K} \mathbf{u} = \mathbf{f}$ two times.

First it reduces the matrix \mathbf{K} to reduced-row-echelon-form (= unit matrix + possible singular columns) to see whether the frame is stable.

If not then the last columns of $\text{rref}(\mathbf{K})$ (row-reduced echelon form, Matlab) are columns which lie in the kernel of \mathbf{K} , in the null-space of \mathbf{K} , [3] p. 267. These rigid-body-movements generate no forces. If these exist, then the system is unstable, is kinematic. The program displays these modes on the screen and notifies the user.

If all is okay the program, then calculates the inverse of \mathbf{K} by applying Gauss' method.

Files in readable form in the folder SDIR... :

| | |
|---------------------|--|
| MATH_KMATRIX.TXT | The original matrix \mathbf{K} , in text-form |
| MATH_RREF.TXT | The reduced-row-echelon form (to detect possible rigid-body-movements) |
| MATH_TRIANGULAR.TXT | The upper triangular matrix |
| MATH_UKINEMATIC.TXT | A possible rigid-body-movement |
| MATH_RHS0X.TXT | The right-hand side of load case X, (read 0 as number null) |
| MATH_SOLOX.TXT | The solution of $\mathbf{K} \mathbf{u} = \mathbf{f}$ in load case X |

If there are more than one null-solutions only the last one is stored and displayed.

20. LITERATURE

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Hartmann F, Jahn P, [Statik und Einflussfunktionen vom modernen Standpunkt aus](#) greatly extended original German edition which preceded the English title [1], now 800+ pages, continuously updated, free download